Mapping and Verification of the Time Ontology in SUMO

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Abstract. Many software systems rely on ontologies for semantic interoperation. However, ontologies which admit unintended models might cause misunderstandings that hinder interoperability because their vocabularies are ambiguously defined. Foundational ontologies, such as SUMO, provide rich characterizations for general concepts that underly every knowledge representation enterprise. Those ontologies are intended to be broadly reused as a reference for semantics. *Ontology verification* is the process by which a theory is checked to rule out unintended models by means of further axiomatization, and characterize missing intended ones. In this paper, we verify the subtheory of core temporal concepts of the SUMO foundational ontology and relate its axiomatization via ontology mapping with other time ontologies, the foundational ontology DOLCE, and the generic ontology PSL. As a result, we propose the addition of some missing axioms that we have identified during our verification task, and the correction of others.

Keywords. ontology verification, ontology mapping, SUMO ontology, DOLCE ontology, PSL ontology

1. Introduction

Automatic applications appealing to ontologies for interoperation are unambiguously integrated only when the models of their shared features are equivalent. However, ontologies admitting unintended models ambiguously characterize their vocabularies, which can generate misunderstandings that hinder interoperability.

Foundational ontologies, also called upper ontologies, characterize the semantics of general concepts that underlay every knowledge representation enterprise. They can be used as oracles for meaning in ontology reconciliation [3], or as the foundational substratum on which new ontologies are developed. Since foundational ontologies are expected to be broadly reused, verifying that they do not have unintended models which can be ruled out with further axiomatization, and that they are not missing intended models, are of paramount interest for the ontology and knowledge representation communities.

Ontology verification is the process by which a theory is checked to rule out its unintended models by means of further axiomatization, and characterize any intended ones which might be missing. In this paper, we apply the definition of ontology verification based on representation theorems that was introduced in [7], which applies to the verification of ontologies axiomatized in first-order logic. It is particularly important to understand the models of upper ontologies. First, it allows us to formally specify the relationships to other upper ontologies, and determine the similarities and differences among them. Second, a characterization of the models of an upper ontology enables us to make the ontological commitments of an upper ontology explicit. Ontology designers that create new domain-specific ontologies by extension of the upper ontology can then be aware of the ontologies that they are using.

After a short review of SUMO [15], we begin the analysis of the axiomatization of its core temporal concepts, which we call *SUMO_Time*. The axioms of SUMO_Time are divided into three subsets, $T_{sumo\_timepoints}$, $T_{sumo\_timeintervals}$, and $T_{sumo\_temporalPart}$, which are explored in the subsequent sections. We show that SUMO_Time admits unintended models and then identify missing axioms which can eliminate these models. After proposing new axioms to extend the original ontology, we demonstrate the verification of the new axioms, and then apply these results to formally prove the relationships between SUMO_Time and other generic ontologies. We have used theorem prover Prover9 [13], and model finder Mace4 in the proofs of all of our results.

Although we provide a characterization of the models of SUMO_Time, and we identify several unintended models of the ontology, we are restricting ourselves in this paper to a logical analysis. We are not debating any philosophical stance implicit in the ontological commitments of a time ontology.

2. The SUMO Foundational Ontology

SUMO [15] is an open source formal foundational ontology axiomatizing, among others, general concepts such as those needed to represent temporal and spatial location, units of measure, objects and processes. As a foundational ontology, it is expected to be used as a global reference for semantics, and its axiomatization reused in the construction of domain and application ontologies for automated reasoning in expressive languages. In addition to the main ontology, which contains about 4000 axioms, SUMO has been extended with a mid-level ontology and a number of domain specific ontologies, all of which account for 20,000 terms and 70,000 axioms in areas such as finance, investment, terrain modeling, distributed computing, and biological viruses. SUMO has been translated into the OWL [11] semantic web language, and has also been mapped to the WordNet lexicon of approximately 100,000 nouns [4] [14], which facilitates the use of the SUMO axiomatization for natural language understanding tasks. We focus on the core axioms of the TEMPORAL subtheory covering the complete axiomatization of time through intervals and points, which we call *SUMO Time*, leaving untouched those axioms concerning other related topics such as dates, measures, durations, and the occurrence of physical entities in time. We plan to study those subtheories in a future work.

The representational language of SUMO is SUO-KIF[1] [18], a dialect of KIF [5]. SUO-KIF is a very expressive language with many-sorted features, whose syntax permits higher-order constructions such as the use of predicates having other predicates, or formulas, as their arguments, and the existence of predicates and functions of variable arity [1]. Mapping axioms from SUO-KIF to conventional first-order logic syntax (for use by theorem provers such as Prover9) requires that we pay close attention to several key features. First, some properties of functions and predicates are characterized by sec-

[1] http://suo.ieee.org/SUO/KIF/suo-kif.html

ond order formulae; for example (`instance before TransitiveRelation`) leads to the transitivity for *before* (see Axiom (3) in Table 1). Second, SUO-KIF uses a rather idiosyncratic approach to order sorted logics [16] where domain and range constructs denote sorts. For example,

```
(domain before 1 TimePoint)
(domain before 2 TimePoint)
```

indicates that the first and second arguments of predicate *before* must be individuals of the extension of predicate *TimePoint*. We will refer to SUMO_Time as the set of SUMO axioms translated into conventional first-order logic syntax[2]. The resulting axioms have been also rewritten using Common Logic.[3]

3. Time Points

We first consider $T_{sumo\_timepoints}$,[4] the subset of the axioms from SUMO_Time that specify an ordering over time points using the *before* relation, given by the axioms shown in Tables 1 and 2 respectively. In this section, we identify two unintended models of $T_{sumo\_timepoints}$, and then propose an extension that eliminates these models.

3.1. Endpoints at Infinity

A distinctive feature of $T_{sumo\_timepoints}$ is that there exist two distinguished time points *NegativeInfinity* (which is before all other time points) and *PositiveInfinity* (which is after all other time points). All other time points are between *NegativeInfinity* and *PositiveInfinity*, and intuitively, there should only exist infinite models of these axioms. However, we have the following:

Proposition 1 $T_{sumo\_timepoints} \not\models (NegativeInfinity \neq PositiveInfinity)$

Proof: Using Mace4, we can construct a model[5] of SUMO_Time in which *NegativeInfinity = PositiveInfinity*. □

Note that in the model that satisfies *NegativeInfinity = PositiveInfinity* there is only one time point. This is admittedly an unusual model, and one may dismiss it as nothing more than a curiosity. Nevertheless, it also means that the axioms do not entail that *NegativeInfinity* is before *PositiveInfinity*, which is quite a fundamental intuition about these two distinguished timepoints.

Interestingly, this issue also appears in other generic ontologies that use the notion of a timeline with points at infinity. In particular, PSL-Core[6] [6] contains a subtheory of time that axiomatizes a linearly ordered set of timepoints with a distinguished timepoint *infneg* which is before all other timepoints and a distinguished timepoint *infpos* which is

[2] `colore.oor.net/sumo/theorems/input/sumo_time.in`
[3] `colore.oor.net/sumo/sumo_time.clif`
[4] `colore.oor.net/sumo/sumo_timepoints`
[5] `colore.oor.net/sumo/theorems/output/ex128-1.model`
[6] `colore.oor.net/psl_core/psl_core.clif`

Table 1. Theory *Tsumo_timepoints*

$$(\forall x,y)\, before(x,y) \rightarrow TimePoint(x) \wedge TimePoint(y) \quad (1)$$

$$(\forall x)\, TimePoint(x) \rightarrow \neg before(x,x) \quad (2)$$

$$(\forall x,y,z)\, before(x,y) \wedge before(y,z) \rightarrow before(x,z) \quad (3)$$

$$TimePoint(PositiveInfinity) \quad (4)$$

$$TimePoint(NegativeInfinity) \quad (5)$$

$$(\forall x)\, TimePoint(x) \wedge \neg(x = PositiveInfinity) \rightarrow before(x, PositiveInfinity) \quad (6)$$

$$(\forall x)\, TimePoint(x) \wedge \neg(x = PositiveInfinity) \rightarrow$$
$$\exists y (TimePoint(y) \wedge temporallyBetween(x,y,PositiveInfinity)) \quad (7)$$

$$(\forall x)\, TimePoint(x) \wedge \neg(x = NegativeInfinity) \rightarrow before(NegativeInfinity, x) \quad (8)$$

$$(\forall x)\, TimePoint(x) \wedge \neg(x = NegativeInfinity) \rightarrow$$
$$\exists y (TimePoint(y) \wedge temporallyBetween(NegativeInfinity, y, x)) \quad (9)$$

$$(\forall x,y,z) temporallyBetween(x,y,z) \leftrightarrow before(x,y) \wedge before(y,z) \quad (10)$$

$$(\forall x,y,z) temporallyBetweenOrEqual(x,y,z) \leftrightarrow$$
$$beforeOrEqual(x,y) \wedge beforeOrEqual(y,z) \quad (11)$$

after all other timepoints, yet $T_{pslcore} \not\models (infneg \neq infpos)$. Similarly, the time ontology with endpoints at infinity $T_{lp\_endpoints}$[7] [10] also admits a finite model in which there is a unique timepoint.

To eliminate such an unintended model, we propose the additional Axiom (21) (see Table 3). Analogous axioms can be added to ontologies PSL-Core and $T_{lp\_endpoints}$. Although it is actually equivalent to $NegativeInfinity \neq PositiveInfinity$, it makes evident the relationship that $NegativeInfinity$ and $PositiveInfinity$ are opposite timepoints along the timeline.

3.2. Linear Orderings on Time Points

If we consider other time ontologies that contain endpoints at infinity, we see that they also satisfy one additional property – the set of timepoints is linearly ordered by the *before* relation. On the other hand, SUMO_Time does not entail the sentence that axiomatizes this property:

[7] colore.oor.net/timepoints/lp_endpoints.clif

Table 2. Theory *Tsumo_timepoints*

$$(\forall x,y) beforeOrEqual(x,y) \rightarrow TimePoint(x) \wedge TimePoint(y) \quad (12)$$

$$(\forall x) TimePoint(x) \rightarrow beforeOrEqual(x,x) \quad (13)$$

$$(\forall x,y) beforeOrEqual(x,y) \wedge beforeOrEqual(y,x) \rightarrow (x = y) \quad (14)$$

$$(\forall x,y,z) beforeOrEqual(x,y) \wedge beforeOrEqual(y,z) \rightarrow beforeOrEqual(x,z) \quad (15)$$

$$(\forall x,y) beforeOrEqual(x,y) \rightarrow before(x,y) \vee (x = y) \quad (16)$$

$$(\forall x,y) before(x,y) \rightarrow beforeOrEqual(x,y) \quad (17)$$

$$(\forall x,y,z) temporallyBetween(x,y,z) \rightarrow TimePoint(x) \wedge TimePoint(y) \wedge TimePoint(z) \quad (18)$$

$$(\forall x,y,z) temporallyBetween(x,y,z) \rightarrow temporallyBetweenOrEqual(x,y,z) \quad (19)$$

$$(\forall x,y,z) temporallyBetweenOrEqual(x,y,z) \rightarrow$$

$$TimePoint(x) \wedge TimePoint(y) \wedge TimePoint(z) \quad (20)$$

Proposition 2 $T_{sumo\_timepoints} \not\models (\forall t_1,t_2)\, TimePoint(t_1) \wedge TimePoint(t_2) \rightarrow (before(t_1,t_2) \vee before(t_2,t_1) \vee (t_1 = t_2)$

Proof: Let $T_{finite\_sumo\_timepoints}$ be the subtheory of $T_{sumo\_timepoints}$ without the axioms that force the existence of infinite sets of time points (e.g. Axioms (7) and (9)). Using Mace4,[8] we can construct a model of $T_{finite\_sumo\_timepoints}$ that contains two timepoints t_0 and t_1 incomparable by the *before* relation (and hence falsifies the sentence). This model can be extended to construct a model of $T_{sumo\_timepoints}$ that also falsifies the sentence by adding linearly ordered sets of timepoints between *NegativeInfinity* and t_0, *NegativeInfinity* and t_1, t_0 and *PositiveInfinity*, and t_1 and *PositiveInfinity*. □

$T_{sumo\_timepoints}$ therefore allows unintended models such as the ones depicted in Figure 1*(a)*, where there exist partially ordered sets of timepoints. Although there do exist time ontologies in which the timepoints are not linearly ordered (e.g. $T_{bp\_ordering}$[9] [10]), in such cases the models are all definably equivalent to semilinear orderings. Since all timepoints are between *NegativeInfinity* and *PositiveInfinity*, the models of $T_{sumo\_timepoints}$ cannot be semilinear orderings. On the other hand, there is no other time ontology that has models in which the comparability graph of the ordering is a directed acyclic graph (as shown in Figure 1-extit(a)). We therefore propose adding Axiom (22) to enforce the linear ordering on timepoints.

[8] `colore.oor.net/sumo/theorems/ex128-2.model`
[9] `colore.oor.net/timepoints/bp_ordering.clif`

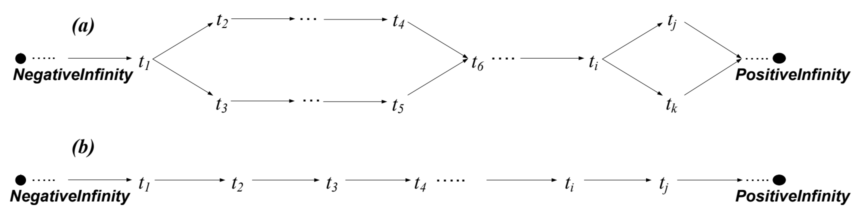

Figure 1. Time models of SUMO with instants connected by relation ***before***.
(a) Nonstandard model where partially ordered branches exist. It is not possible to know what the order is among all parts of an event that started to occur at instant t_1 and finished at instant t_6, even though the instants at which those parts occurred are known. For example, we can not know if what occurred at t_4 occurred before, after, or simultaneously to what occurred at t_5 because those points are not connected by the transitive closure of the ordering relation ***before***.
(b) Linear time model where every instant is connected in a unique *timeline* by the transitive closure of the ***before*** relation.

3.3. $T_{sumo\_ordered\_timepoints}$

Incorporating these changes, we have $T_{sumo\_ordered\_timepoints}$ is the set of axioms[10] in Table 1 and Table 3.

Note that we have given a conservative definition to the *beforeOrEqual* relation, which is not the case in $T_{sumo\_timepoints}$ of the original SUMO (see the subtheory in Table 2). On the other hand, because *before* is a subrelation of *beforeOrEqual* in SUMO, we should intuitively be able to derive

$$(\forall x,y)\,(before(x,y) \vee (x=y) \rightarrow beforeOrEqual(x,y)$$

yet this sentence cannot be entailed from $T_{sumo\_timepoints}$.

Table 3. Proposed additional axioms for $T_{sumo\_ordered\_timepoints}$

$$before(NegativeInfinity, PositiveInfinity) \quad (21)$$

$$(\forall t_1,t_2)\, TimePoint(t_1) \wedge TimePoint(t_2) \rightarrow (before(t_1,t_2) \vee before(t_2,t_1) \vee (t_1 = t_2) \quad (22)$$

$$(\forall x,y)\, beforeOrEqual(x,y) \leftrightarrow$$

$$before(x,y) \vee ((x=y) \wedge TimePoint(x) \wedge TimePoint(y)) \quad (23)$$

Proposition 3 $T_{sumo\_ordered\_timepoints} \models T_{sumo\_timepoints}$

Proof: The proofs[11] were generated by Prover9. □

Thus, we have proposed $T_{sumo\_ordered\_timepoints}$, a nonconservative extension of $T_{sumo\_timepoints}$ that eliminates two classes of unintended models - a trivial model with a

[10] `colore.oor.net/sumo_timepoints/sumo_ordered_timepoints.clif`
[11] `colore.oor.net/sumo/theorems/timepoints_entails/`

unique time point, and models in which time points are not linearly ordered by the *before* relation.

4. Temporal Mereology

The remaining axioms in SUMO-Time extend the ontology by adding time intervals to the domain. Besides axiomatizing the relationships between timepoints and timeintervals, SUMO-Time also specifies a mereology over time intervals. In this section, we explore this mereology; we identify a missing axiom, and use this axiom to show that the axiomatization of the mereology over time intervals can be considered to be a definitional extension of a subtheory of SUMO-Time.

4.1. TimePoints and TimeIntervals

In addition to an ordering over timepoints, SUMO-Time also contains a subtheory $T_{sumo\_timeintervals}$[12] (shown in Table 4) that introduces time intervals and their relationship to timepoints. The functions $BeginFn$ and $EndFn$ map a time interval to its beginning and ending timepoints, respectively, The function $TimeIntervalFn$ maps a pair of timepoints to the time interval for which they are the begin and end.

From a visual inspection of $T_{sumo\_timeintervals}$, it is easy to see that there is a close relationship to other time ontologies from [10] In the next section, we will make this relationship precise.

4.2. Problems with the Existing Axiomatization of $temporalPart$

We first consider $T_{sumo\_temporalPart}$[13], the subset of the axioms from SUMO-Time that specify a mereology over time positions (i.e. both timepoints and timeintervals) using the $temporalPart$ relation. Besides a set of axioms that explicitly specify that $temporalPart$ is a partial ordering, $T_{sumo\_temporalPart}$ also contains a set of axioms that specify the relationships between $temporalPart$ and Allen's interval relations[14].

Problems arise when we consider the $temporalPart$ relation and timepoints – there exist models in which timepoints contain timeintervals as temporal parts:

Proposition 4 $T_{sumo\_timepoints} \cup T_{sumo\_timeintervals} \cup T_{sumo\_temporalPart} \not\models \neg(\exists x,y)\, TimePoint(x) \wedge TimeInterval(y) \wedge temporalPart(y,x)$

Proof: Using Mace4[15], one can construct a model of $T_{finite\_sumo\_timepoints} \cup T_{sumo\_timeintervals} \cup T_{sumo\_temporalPart}$ that falsifies the sentence. It is easy to see that this model can be extended to construct a model of $T_{sumo\_timepoints} \cup T_{sumo\_timeintervals} \cup T_{sumo\_temporalPart}$ that also falsifies the sentence. □

[12] `colore.oor.net/sumo_timeintervals/sumo_timeintervals.clif`

[13] `colore.oor.net/sumo/sumo_temporalPart`

[14] Within the axioms of SUMO-Time, the *during* relation is not given a conservative definition, as is the case with the other interval relations. We considered this to be a typographical error in the axioms, rather than an ontological commitment, and hence we assume that *during* is supposed to have a conservative definition.

[15] `colore.oor.net/sumo/theorems/ex128-3.model`

Table 4. Theory *Tsumo_timeintervals*

$$(\forall x,y,z)TimePoint(x) \wedge TimePoint(y) \wedge TimeInterval(z) \wedge$$

$$(TimeIntervalFn(x,y) = z) \rightarrow (BeginFn(z) = x) \wedge (EndFn(z) = y) \tag{24}$$

$$(\forall x,y,z,t)TimeInterval(x) \wedge TimeInterval(y) \wedge TimePoint(z) \wedge TimePoint(t) \rightarrow$$

$$((BeginFn(x) = z) \wedge (BeginFn(y) = z) \wedge (EndFn(x) = t) \wedge (EndFn(y) = t) \rightarrow (x = y) \tag{25}$$

$$(\forall x,y,z)TimeInterval(x) \wedge TimePoint(y) \wedge TimePoint(z) \rightarrow$$

$$(BeginFn(x) = y) \wedge (EndFn(x) = z) \rightarrow before(y,z) \tag{26}$$

$$(\forall x)TimeInterval(x) \rightarrow TimePoint(BeginFn(x)) \wedge TimePoint(EndFn(x)) \tag{27}$$

$$(\forall x,y)TimePoint(x) \wedge TimePoint(y) \wedge before(x,y) \rightarrow TimeInterval(TimeIntervalFn(x,y)) \tag{28}$$

$$(\forall x)TimePosition(x) \leftrightarrow TimeInterval(x) \vee TimePoint(x) \tag{29}$$

$$(\forall x)TimeInterval(x) \rightarrow \neg TimePoint(x) \tag{30}$$

We can eliminate models in which timepoints contain timeintervals as temporal parts by adding the axiom that enforces the condition that the only temporal part of a timepoint is the timepoint itself. In addition to eliminating this class of unintended models, such an axiom allows us to axiomatize the *temporalPart* relation as a definitional extension of the ontology:

Definition 1 *$T_{sumo\_temporal\_mereology}$ is the set of sentences in Table 5.*

Table 5. Theory *Tsumo_temporal_mereology*

$$(\forall x,y)\, temporalPart(x,y) \rightarrow TimePosition(x) \wedge TimePosition(y) \tag{31}$$

$$(\forall x,y)\, TimePosition(x) \wedge TimePoint(y) \rightarrow (temporalPart(x,y) \leftrightarrow (x = y)) \tag{32}$$

$$(\forall x,y)\, TimePoint(x) \wedge TimeInterval(y) \rightarrow$$

$$(temporalPart(x,y) \leftrightarrow temporallyBetweenOrEqual(BeginFn(y),x,EndFn(y))) \tag{33}$$

$$(\forall x,y)\, TimeInterval(x) \wedge TimeInterval(y) \rightarrow (temporalPart(x,y) \leftrightarrow$$

$$beforeOrEqual(BeginFn(y),BeginFn(x)) \wedge beforeOrEqual(EndFn(x),EndFn(y))) \tag{34}$$

Theorem 1 *$T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals} \cup T_{sumo\_temporal\_mereology}$ is a definitional extension of $T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$.*

Proof: Using Prover9[16] we can show that $T_{sumo\_temporal\_mereology}$ is logically equivalent to a conservative definition for the *temporalPart* relation. □

Another way of looking at this result, is that we can specify a definitional extension of $T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$ which is stronger than the original axiomatization of SUMO-Time.

Proposition 5 $T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals} \cup T_{sumo\_temporal\_mereology} \models T_{sumo\_temporalPart}$

Proof: Proofs [17]. were generated by Prover9. □

This approach greatly simplifies SUMO_Time in terms of its axiomatization as well as its conceptualization.

Finding a minimal set of axioms may seem to be an intellectual diversion, or even a fetish. For example, it is difficult to see the benefit of finding the minimal set of equations for Boolean lattices once we are confident that the axioms do in fact axiomatize Boolean lattices. Nevertheless, by identifying a minimal set of axioms which are sufficient for axiomatizing a class of structures, we are in a better position to gaining insights into the fundamental ontological commitments of an ontology. We can distinguish the axioms that capture the basic ontological commitments of SUMO-Time from those sentences which are logical consequences. In particular, we can see that the real ontological commitments are in the axiomatization of the relationship between timepoints and timeintervals.

It is easy to see that $T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals} \cup T_{sumo\_time\_mereology}$ is a nonconservative extension of T_{sumo_time}. The question remains - have we eliminated *all* of the unintended models? What exactly are the models of the extended ontology? We address this question in the following section.

5. Verification of the Extension of SUMO_Time

Verification is concerned with the relationship between the intended models of an ontology and the models of the axiomatization of the ontology. In particular, we want to characterize the models of an ontology up to isomorphism[18] and determine whether or not these models are equivalent to the intended models of the ontology. This relationship between the intended models and the models of the axiomatization plays a key role in the application of ontologies in areas such as semantic integration and decision support.

Unfortunately, it can be quite difficult to characterize the models of an ontology up to isomorphism. Ideally, since the classes of structures that are isomorphic to an ontology's models often have their own axiomatizations, we should be able to reuse the characterizations of these other structures. The key to this endeavour is the notion of logical synonymy:

[16] `colore.oor.net/sumo/theorems/temporalPart_definitional/`
[17] `colore.oor.net/sumo/theorems/temporalPart_entails/`
[18] Isomorphic structures have the same model-theoretic properties.

Definition 2 *Two theories T_1 and T_2 are synonymous iff there exist two sets of translation definitions Δ and Π, respectively from T_1 to T_2 and from T_2 to T_1, such that $T_1 \cup \Pi$ is logically equivalent to $T_2 \cup \Delta$.*

By the results in [17], there is a bijection on the sets of models for synonymous theories. We can therefore characterize the models of the ontology being verified by demonstrating that the ontology is synonymous with a logical theory whose models we understand. In particular, using the approach taken in [9] to show the following:

Theorem 2 *$T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$ is logically synonymous with $T_{bounded\_linear\_ordering}$[19] $\cup$ $T_{strict\_graphical}$[20].*

Proof: Given the set of translation definitions[21] Δ shown in Table 6, using Prover9, we have shown that $T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals} \cup \Delta \models T_{bounded\_linear\_ordering} \cup T_{strict\_graphical}$. In addition, given the set of translation definitions Π, shown in Table 7, we have proved that $T_{bounded\_linear\_ordering} \cup T_{strict\_graphical} \cup \Pi \models T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$. Finally, we can use Prover9 to show that $T_{bounded\_linear\_ordering} \cup T_{strict\_graphical} \cup \Delta \models \Pi$, and $T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals} \cup \Pi \models \Delta$. Therefore, the theories are synonymous. □

Table 6. Translation Definitions Δ

$$(\forall x)\, point(x) \equiv TimePoint(x)$$

$$(\forall x)\, line(x) \equiv TimeInterval(x)$$

$$(\forall x,y)\, in(x,y) \equiv (begin(y,x) \vee end(y,x) \vee (x=y))$$

$$(\forall x,y)\, lt(x,y) \equiv before(x,y)$$

We can use this result to characterize the models of $T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$:

Corollary 1 *$\mathscr{M} \in Mod(T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals})$ iff*

1. *$\mathscr{M} \cong \langle P \cup G, \mathbf{lt}, \mathbf{in}^{\mathbf{G}} \rangle$, where*
   (a) *$\langle P, \mathbf{lt} \rangle$ is a linear ordering*

[19] `colore.oor.net/orderings/bounded_linear_ordering.clif`

[20] `colore.oor.net/bipartite_incidence/strict_graphical.clif`

[21] Function symbols that require restricted quantification of arguments in every sentence because they represent partial functions, such as SUMO *BeginFn*, *EndFn*, and *TimeIntervalFn*, can not be mapped into relations. In order to make map Π possible, we have characterized predicates *begin*, *end*, and *interval* with the semantics of SUMO function symbols *BeginFn*, *EndFn*, and *TimeIntervalFn*, and produced the corresponding syntactic translation of every sentence of theory *Tsumo_timeintervals* into the new representation. We have used the same representation for map Δ.

Table 7. Translation Definitions Π

$$(\forall x)\, TimePoint(x) \equiv point(x)$$

$$(\forall x)\, TimeInterval(x) \equiv line(x)$$

$$(\forall x,y)\, begin(y,x) \equiv (line(y) \wedge point(x) \wedge (in(x,y) \wedge (\forall z)\, point(z) \wedge in(z,y) \rightarrow leq(x,z))$$

$$(\forall x,y)\, end(y,x) \equiv (line(y) \wedge point(x) \wedge (in(x,y) \wedge ((\forall z)\, point(z) \wedge in(z,y) \rightarrow leq(z,x))$$

$$((\forall x,y,z)\, interval(x,y,z) \equiv point(x) \wedge point(y) \wedge line(z) \wedge in(x,z) \wedge in(y,z) \wedge lt(x,y))$$

$$(\forall x,y)\, before(x,y) \equiv lt(x,y)$$

(b) $\langle P, G, \mathbf{in^G} \rangle$ *is a strict graphical incidence structure*[22]..

2. $\langle \mathbf{t} \rangle \in$ **TimePoint** *iff* $\mathbf{t} \in P$*;*
3. $\langle \mathbf{i} \rangle \in$ **TimeInterval** *iff* $\mathbf{i} \in G$*;*
4. $\mathbf{BeginFn(i) = t}$ *iff* $\langle \mathbf{t}, \mathbf{i} \rangle \in \mathbf{in^G}$ *and for any* $\mathbf{t'} \in P$ *such that* $\langle \mathbf{t'}, \mathbf{i} \rangle \in \mathbf{in^G}$, *we have* $\langle \mathbf{t}, \mathbf{t'} \rangle \in \mathbf{lt}$.
5. $\mathbf{EndFn(i) = t}$ *iff* $\langle \mathbf{t}, \mathbf{i} \rangle \in \mathbf{in^G}$ *and for any* $\mathbf{t'} \in P$ *such that* $\langle \mathbf{t'}, \mathbf{i} \rangle \in \mathbf{in^G}$, *we have* $\langle \mathbf{t'}, \mathbf{t} \rangle \in \mathbf{lt}$.
6. $\mathbf{TimeIntervalFn(t_1, t_2) = i}$ *iff* $\langle \mathbf{t_1}, \mathbf{i} \rangle$, $\langle \mathbf{t_2}, \mathbf{i} \rangle \in \mathbf{in^G}$*;*
7. $\langle \mathbf{t_1}, \mathbf{t_2} \rangle \in$ **before** *iff* $\langle \mathbf{t_1}, \mathbf{t_2} \rangle \in \mathbf{lt}$.

6. Relationship to Other Ontologies

The verification of $T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$ does not only provide us with a representation theorem for its models, but it also enables us to formally specify the relationships to other ontologies by reusing the verification of these other ontologies.

6.1. Combined Time Ontologies

As noted in Section 3, an inspection of the signatures of SUMO_Time and the ontology $T_{endpoints}$ indicates that there should be a close relationship between the two ontologies. In fact, we have the following:

Theorem 3 *$T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$ is synonymous with $T_{interval\_with\_endpoints}$.*

Proof: Let Υ be the set of translation definitions shown in Table 8, Using Prover9, we have shown that

[22] A **strict graphical incidence structure** is a tuple $\mathbb{G} = \langle X, Y, \mathbf{in^G} \rangle$ where $X \cap Y = \emptyset$, $\mathbf{in^G} \subseteq (X \times Y)$, and elements of $\mathbb{G}$ that are related by **in** are called incident. All elements of Y are incident with exactly two elements of X, and for each pair of points $\mathbf{p}, \mathbf{q} \in X$ there exists a unique element in Y that is incident with both $\mathbf{p}$ and $\mathbf{q}$.

$$T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals} \cup \Upsilon \models T_{interval\_with\_endpoints}$$
$$T_{interval\_with\_endpoints} \cup \Upsilon \models T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$$

Table 8. Translation Definitions Υ

$$(\forall x)\, TimePoint(x) \equiv timepoint(x)$$

$$(\forall x)\, TimeInterval(x) \equiv timeinterval(x)$$

$$(\forall x,y)\, (BeginFn(x) = y) \equiv (beginof(x) = y)$$

$$(\forall x,y)\, (EndFn(x) = y) \equiv (endof(x) = y)$$

$$(\forall x,y,z)\, (TimeIntervalFn(x,y) = z) \equiv (between(x,y) = z)$$

□

Since $T_{interval\_with\_endpoints}$ is inconsistent with $T_{endpoints}$, it follows that $T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$ cannot interpret $T_{endpoints}$. Nevertheless, we can still specify the relationship between these theories. We can use the notion of generalized similarity (introduced in [8]) to compare two theories that are in different hierarchies of an ontology repository and thereby identify the maximal shared subtheory between them.

Theorem 4 *The generalized similarity of* $T_{endpoints}$ *by* $T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$ *is* $T_{finite\_endpoints}$[23].

Proof: We have already seen that $T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$ is synonymous with $T_{interval\_with\_endpoints}$[24]. Since $T_{finite\_endpoints}$ is the similarity of $T_{endpoints}$ and $T_{interval\_with\_endpoints}$, by the definition we have that the generalized similarity of $T_{endpoints}$ by $T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$ is $T_{finite\_endpoints}$. □

This result tells us that the difference between SUMO_Time and $T_{endpoints}$ lies in the underlying ordering on time points rather than in the relationship between time points and time intervals.

6.2. Generic and Upper Ontologies

Theories such as $T_{endpoints}$ that axiomatize the relationships between time points and time intervals appear in several other ontologies, and we can leverage the verification of SUMO_Time to specify mappings to these generic and upper ontologies.

[23] `color.oor.net/combined_time/finite_endpoints`
[24] `colore.oor.net/combined_time/interval_with_endpoints.clif`

6.2.1. PSL

We earlier noted that the PSL-Core ontology contains a time ontology that axiomatized a linear ordering over time points that included time points at infinity. Although the original PSL-Core ontology did not contain time intervals, the work of [2] extended it by importing $T_{interval\_with\_endpoints}$. Using Theorem 3 therefore gives us the following:

Theorem 5 *$T_{interval\_psl\_core}$ faithfully interprets $T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$.*

In a sense, the revised axiomatization of SUMO_Time could be seen as the natural time ontology for $T_{interval\_psl\_core}$[25].

6.2.2. DOLCE

The upper ontology DOLCE [12] axiomatizes the mereology on time intervals $T_{dolce\_time\_mereology}$.[26]. The verification of the revised SUMO_Time not only enables us to specify the relationship to DOLCE, it also allows us to identify which mereology on time intervals is definable in models of $T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$.

Theorem 6 *$T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$ interprets $T_{dolce\_time\_mereology}$.*

Proof: By the results in [2], $T_{interval\_with\_endpoints}$ interprets the time interval ontology $T_{cem\_periods}$,[27] which in turn faithfully interprets $T_{dolce\_time\_mereology}$. Together with Theorem 3, we know that $T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$ interprets $T_{dolce\_time\_mereology}$. □

We can therefore conclude that a complete extensional mereology is definable over time intervals in models of $T_{sumo\_ordered\_timepoints} \cup T_{sumo\_timeintervals}$.

7. Summary

SUMO is a widely used upper ontology, yet there is not a full understanding of its metalogical properties. Following the approach of [8], in which an upper ontology is considered to be composed of generic ontologies, we have begun the analysis of SUMO by the verification of the time ontology (SUMO_Time) within SUMO. We identified three classes of unintended models of SUMO_Time – models with a unique timepoint, models with partially ordered sets of time points, and models in which time intervals are temporal parts of time points. We proposed three new axioms that eliminate these unintended models, and then verified the extended theory, characterizing its models up to elementary equivalence. An interesting outcome was a restructuring of SUMO_Time as the definitional extension of two modules, thus greatly simplifying the axiomatization. Finally, we specified the metalogical relationships between various time ontologies and the new axiomatization of SUMO_Time.

[25] `colore.oor.net/interval_psl/interval_psl_core.clif`
[26] `colore.oor.net/dolce_time_mereology/dolce_time_mereology.clif`
[27] `colore.oor.net/periods/cem_periods.clif`

The approach taken in this paper will be extended to the remaining generic ontologies within SUMO – dates and duration, mereotopology, process, measure, objects, and qualities.

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